TCC 2022

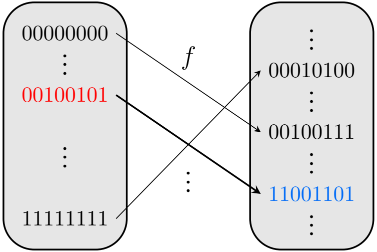
The Parallel Reversible Pebbling Game: Analyzing the Post-Quantum Security of iMHFs

Jeremiah Blocki, Blake Holman, and Seunghoon Lee

November 7, 2022

Motivation: Quantum Pre-Image Attack

Problem. Given a function $f : \{0,1\}^n \to \{0,1\}^n$ and a target output y, find an input $x \in \{0,1\}^n$ such that $y = f(x)$.



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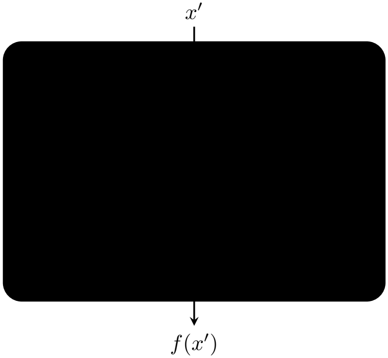

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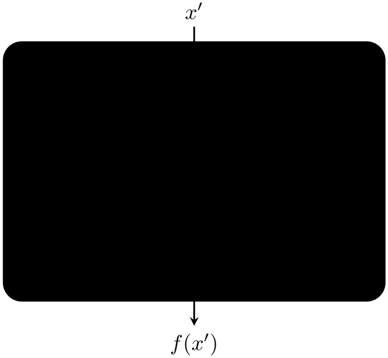

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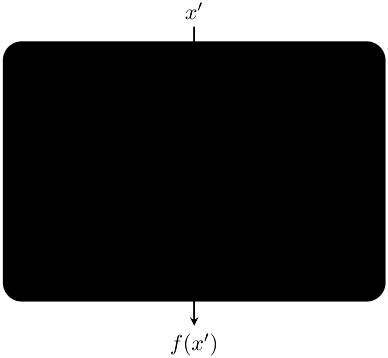

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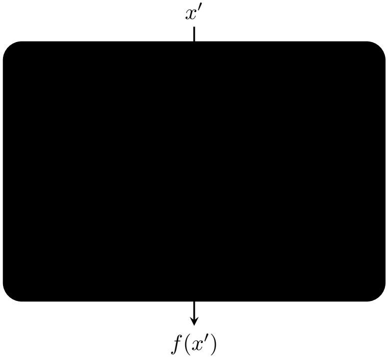

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- What is the full cost of a quantum pre-image attack?

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The full cost of a quantum pre-image attack is defined as the *space-time cost* (ST-cost), i.e.,

$$(\text{space}(C)) \times (\text{time}(C)),$$

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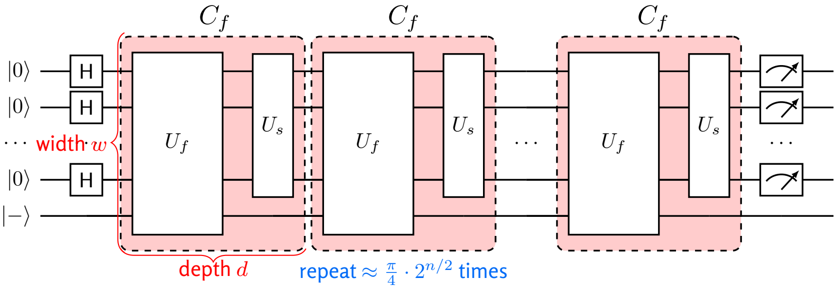

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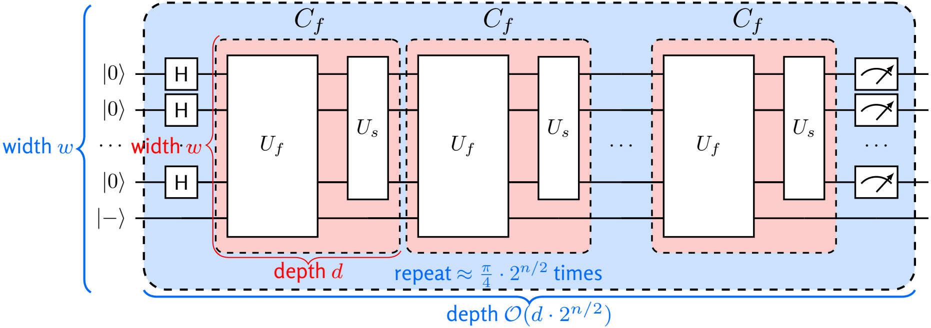

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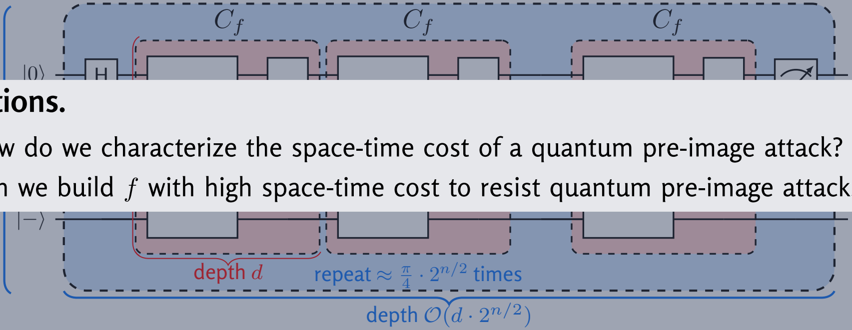

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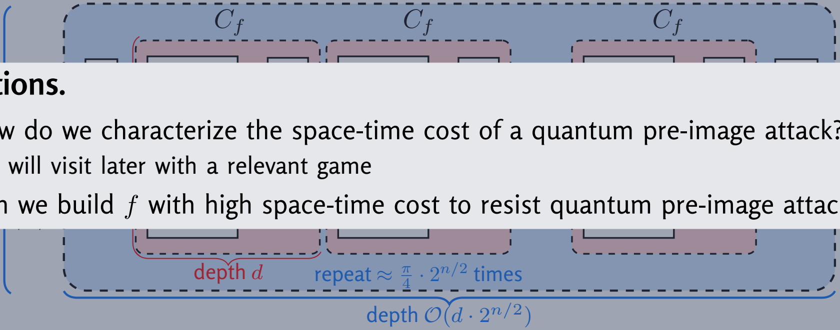

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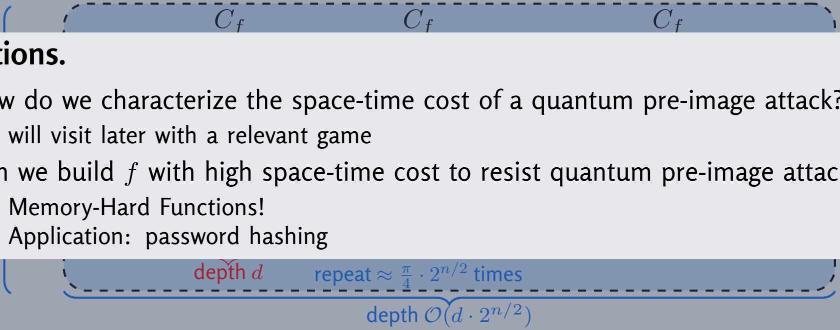

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  - Memory-Hard Functions!
  - Application: password hashing

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Data-Independent Memory-Hard Function (iMHF)

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- $H : \{0,1\}^{2k} \rightarrow \{0,1\}^k$ (Random Oracle)
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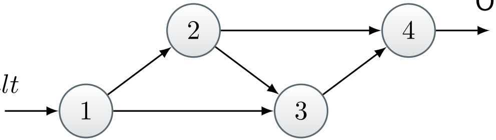

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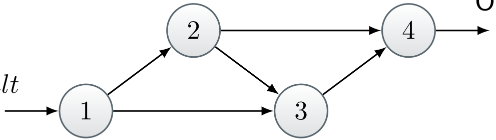

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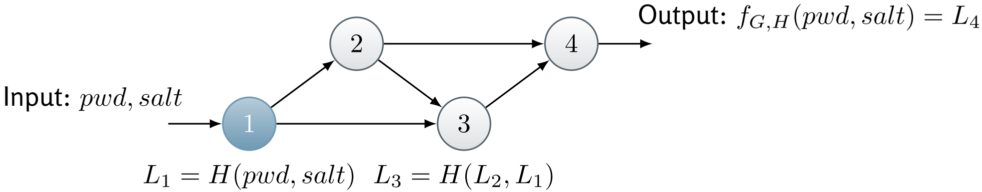

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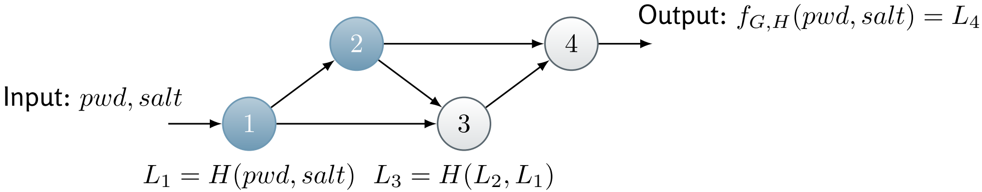

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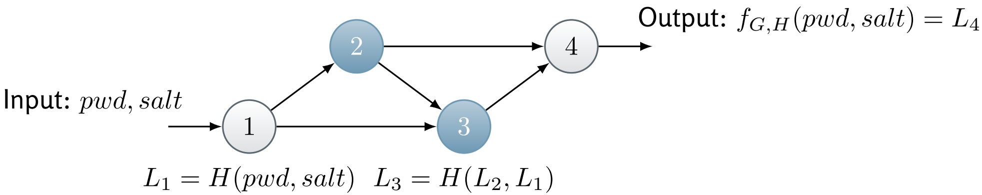

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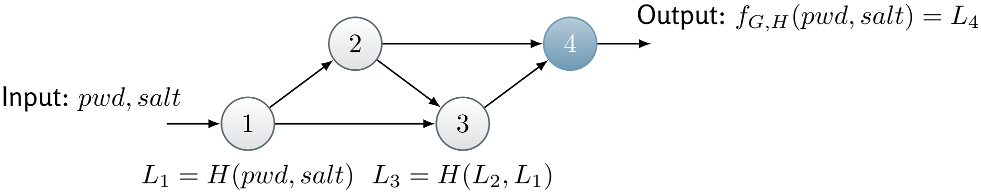

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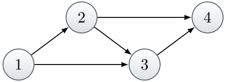

A pebbling $P = (P_1 = \{1\}, P_2 = \{1,2\}, P_3 = \{2,3\}, P_4 = \{4\})$

Space-Time (ST) Complexity

- $\mathsf{ST}(P) = (\text{time}) \times (\text{max space})$, and $\mathsf{ST}(G) = \min_P \mathsf{ST}(P)$
- For above example, we have

$$\mathsf{ST}(P) = 4 \times 2 = 8$$

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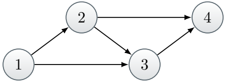

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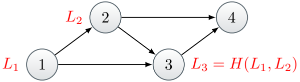

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Why?

- Quantum circuits must be *reversible*
- $P_3 = \{2,3\} \to P_4 = \{4\}$: not a *reversible* transition
- **Quantum Uncomputation** in the QROM:

$$|x, y\rangle \xmapsto{H} |x, y \oplus H(x)\rangle$$

$$\begin{aligned}\therefore |(L_1, L_2), L_3\rangle &\xmapsto{H} |(L_1, L_2), L_3 \oplus H(L_1, L_2)\rangle \\ &= |(L_1, L_2), 0^k\rangle\end{aligned}$$

$\therefore$ to remove a pebble from node 3 using uncomputation, we need have needed pebbles on nodes 1 and 2

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- Prior work [Ben89, LV96, Krá01, MSR$^+$19] introduced *sequential* reversible computation /reversible pebbling game
  - Added more constraints to capture reversible transitions by quantum uncomputation
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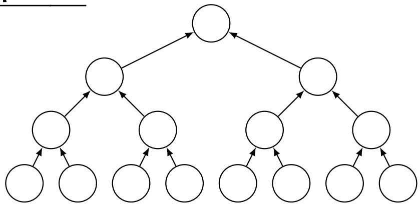

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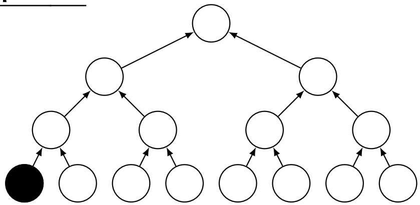

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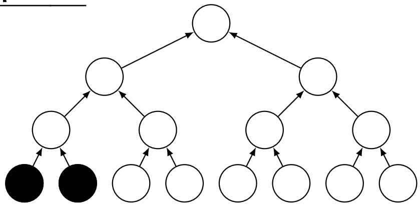

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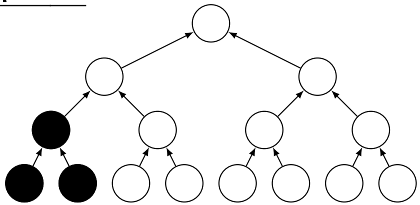

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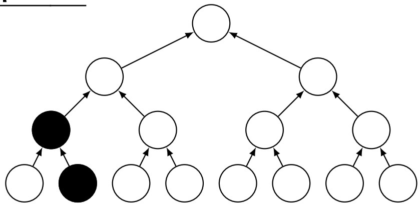

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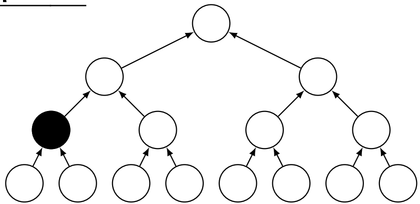

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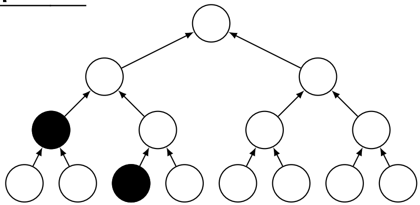

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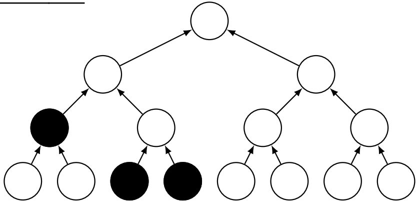

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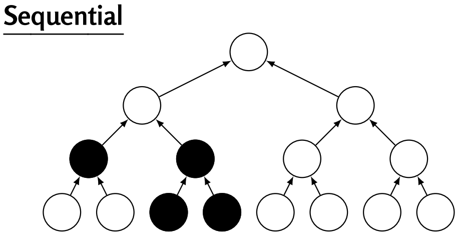

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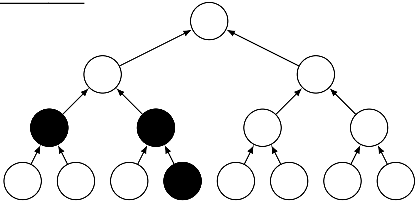

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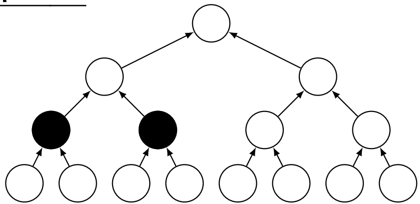

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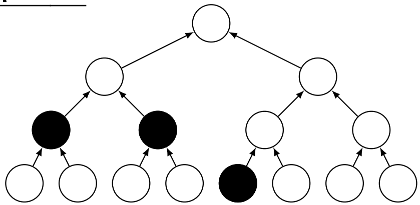

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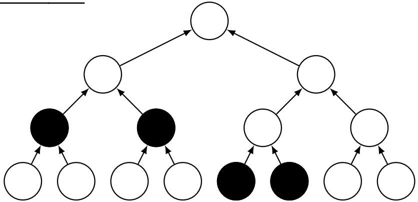

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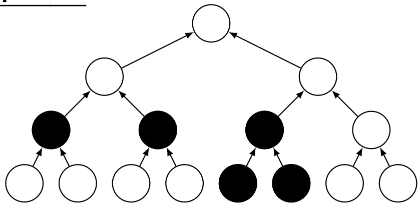

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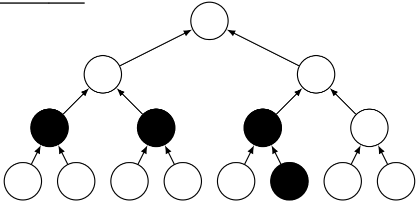

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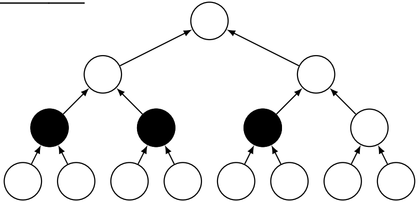

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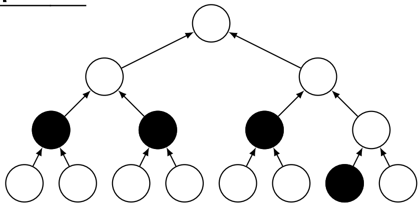

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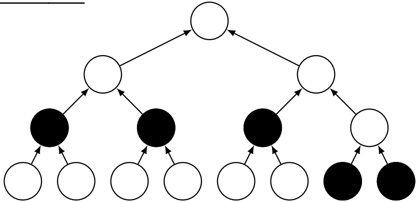

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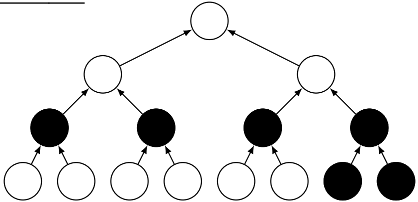

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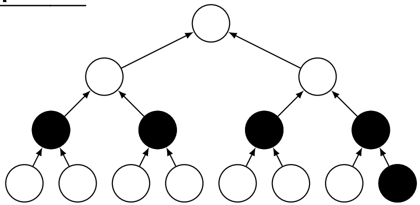

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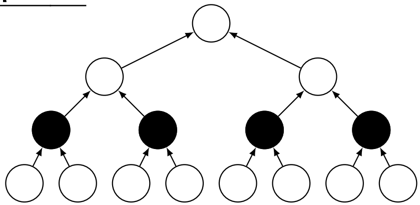

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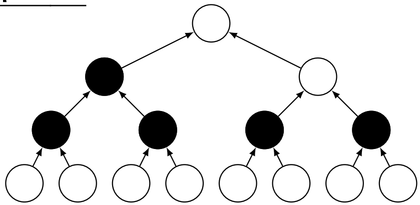

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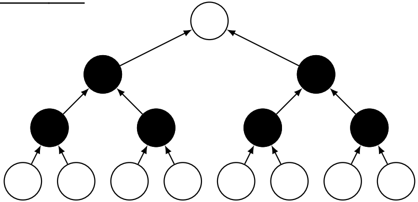

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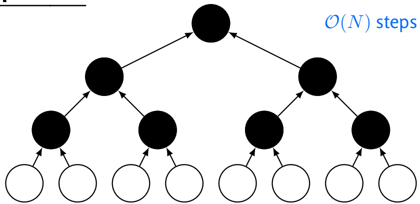

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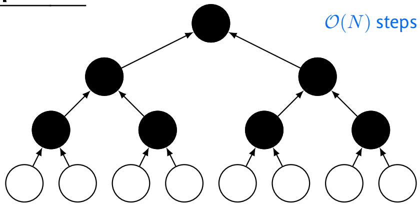

$\mathcal{O}(N)$ steps

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$$\Rightarrow \text{(ST-Cost)} = \text{(space)} \times \text{(time)}$$
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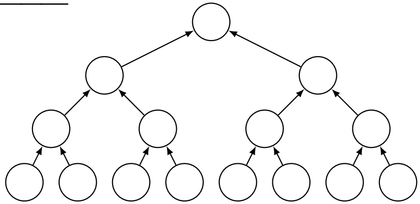

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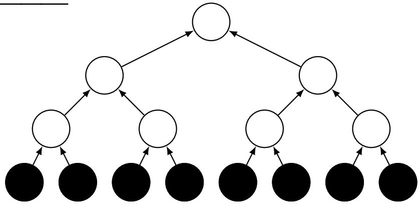

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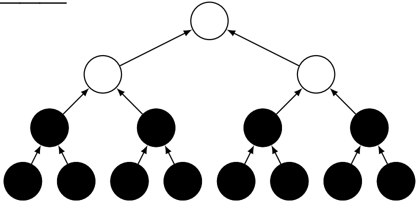

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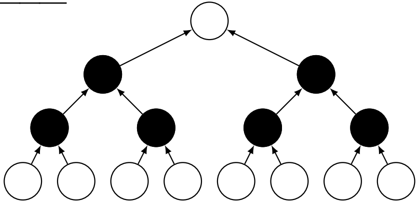

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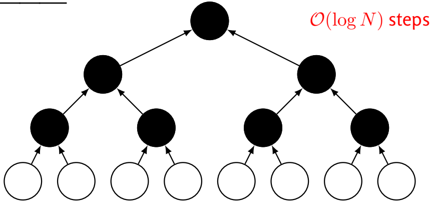

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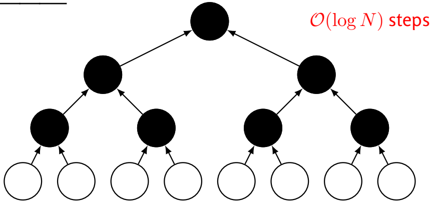

Sequential Reversible Pebbling:

$\Rightarrow$ (ST-Cost) $=$ (space) $\times$ (time) $= 7 \times 23 = 161$

Analyzing the Quantum Circuit:

$\Rightarrow$ (ST-Cost) $= 12 \times 4 = 48$

$\Rightarrow$ the time cost can be decreased from $\mathcal{O}(N)$ to $\mathcal{O}(\log N)$

Key Research Question

Back to Our First Question

How to characterize the space-time cost of a quantum circuit $C_{f_{G,H}}$ for a data-independent Memory-Hard Function $f_{G,H}$?

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Partial Answer: *The Parallel Reversible Pebbling Game*
& Study some attacks against iMHFs in this pebbling model

Definition: Parallel Reversible Pebbling Game

A *parallel reversible pebbling* $P = (P_0, \ldots, P_t)$ is a sequence of pebbling configurations with the conditions (same as classical):

1. start with no pebbles (i.e., $P_0 = \varnothing$) and end with target nodes T (i.e., $T \subseteq P_t$) $^{(\star)}$,
2. a new pebble can be added only if its parents were previously pebbled, and

the following *additional* conditions:

Condition 3. (Quantum No-Deletion)

a pebble *can be deleted* only if all of its parents were previously pebbled

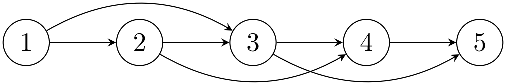
Condition 4. (Quantum Reversibility)

we *must keep the pebble* if a pebble was required to generate new pebbles (or delete pebbles)

$(\star)$ we can make this condition strict, i.e., $P_t = T$. See the paper for detail.

Example: A Parallel Pebbling

Classical vs. Reversible



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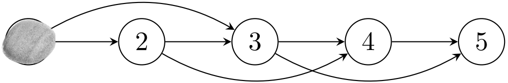

Classical

Round 1 ● ○ ○ ○ ○

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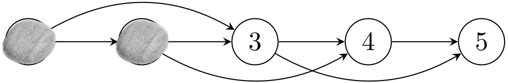

	Classical				
Round 1	●	○	○	○	○
Round 2	●	●	○	○	○

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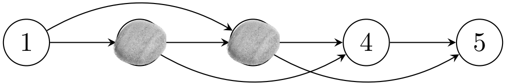

	Classical				
Round 1	●	○	○	○	○
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Round 3	○	●	●	○	○

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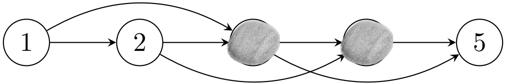

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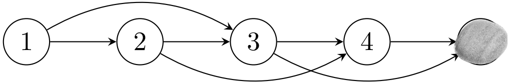

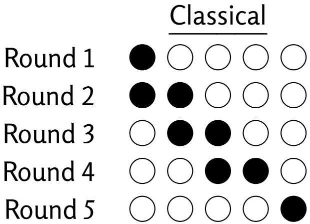

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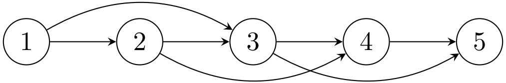

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Round 1	●	○	○	○	○
Round 2	●	●	○	○	○
Round 3	○	●	●	○	○
Round 4	○	$\boxed{\circ}$	●	●	○
Round 5	○	$\boxed{\circ}$	$\boxed{\circ}$	○	●

$\boxed{}$: illegal in a reversible pebbling by Condition 3 (Quantum No-Deletion)

– cannot remove pebble since not all parents were pebbled

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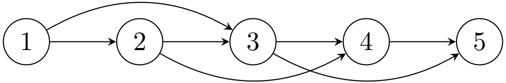

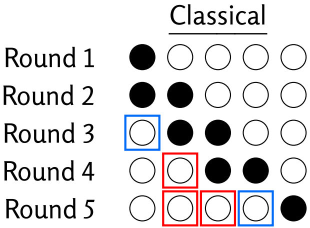

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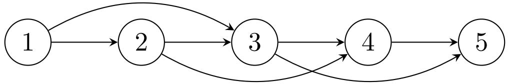

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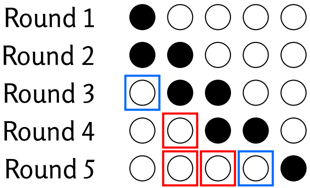

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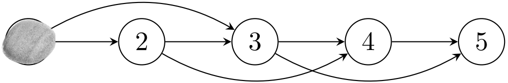

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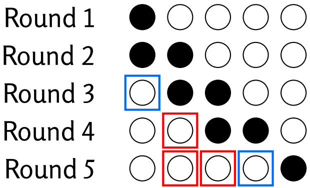
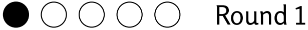


Reversible



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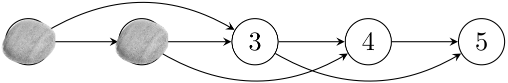

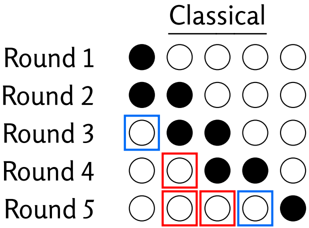

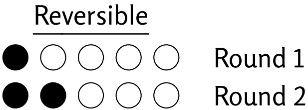

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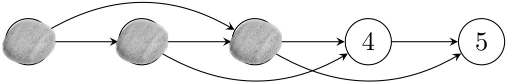
Example: A Parallel Pebbling

Classical vs. Reversible



Classical

Round 1	●	○	○	○	○
Round 2	●	●	○	○	○
Round 3	○	●	●	○	○
Round 4	○	○	●	●	○
Round 5	○	○	○	○	●

Reversible

●	○	○	○	○	Round 1
●	●	○	○	○	Round 2
●	●	●	○	○	Round 3

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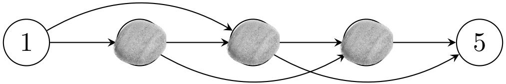

Classical

Round 1	●	○	○	○	○
Round 2	●	●	○	○	○
Round 3	○	●	●	○	○
Round 4	○	○	●	●	○
Round 5	○	○	○	○	●

Reversible

●	○	○	○	○	Round 1
●	●	○	○	○	Round 2
●	●	●	○	○	Round 3
○	●	●	●	○	Round 4

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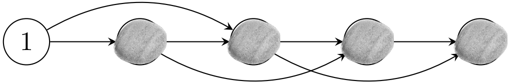

Classical

Round 1	●	○	○	○	○
Round 2	●	●	○	○	○
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Round 5	○	○	○	○	●

Reversible

●	○	○	○	○	Round 1
●	●	○	○	○	Round 2
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○	●	●	●	○	Round 4
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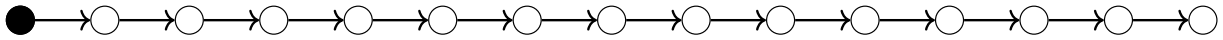

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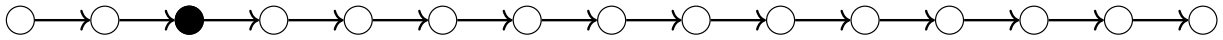

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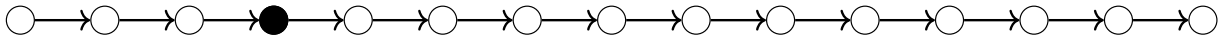

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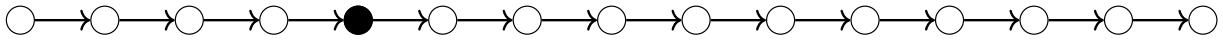

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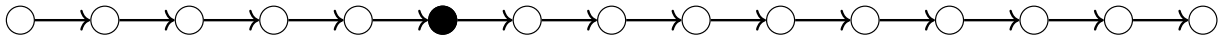

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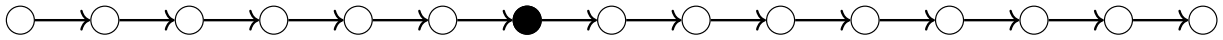

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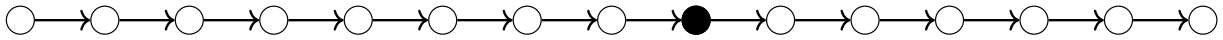

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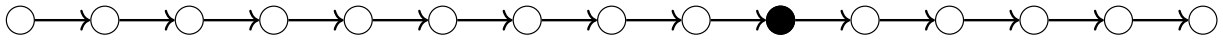

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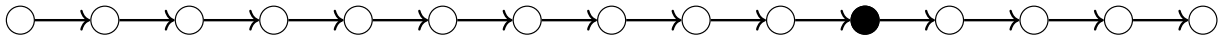

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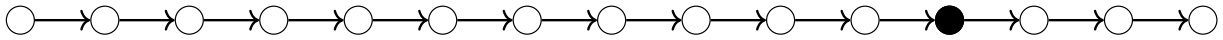

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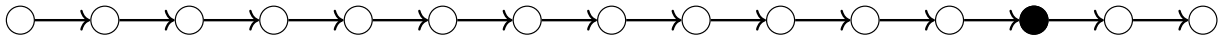

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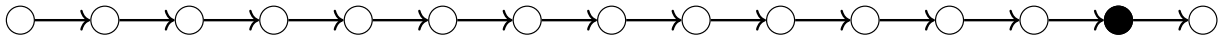

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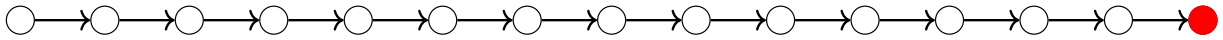

(ST-Cost) $= N$

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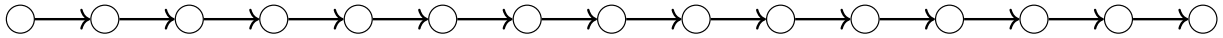

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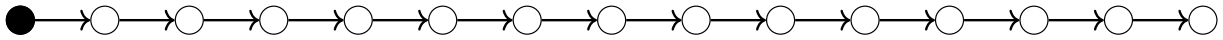

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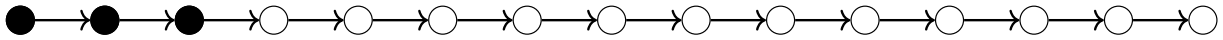
Naïve Reversible Pebbling 

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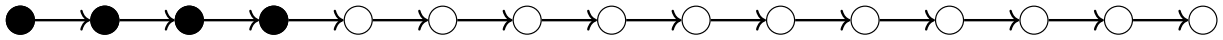
Naïve Reversible Pebbling 

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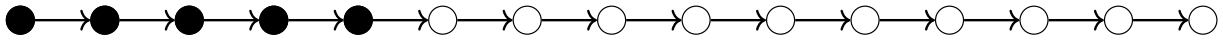
Naïve Reversible Pebbling 

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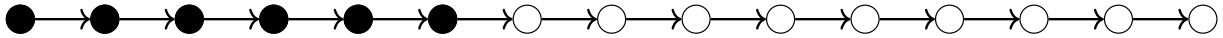

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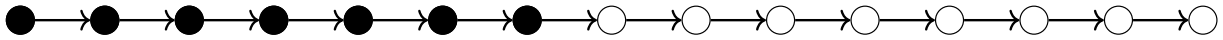

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$(\text{ST-Cost}) = N^2$

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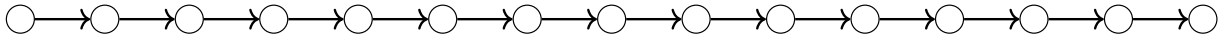

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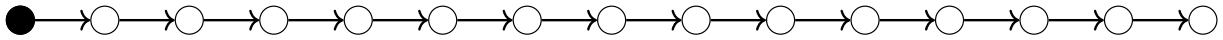

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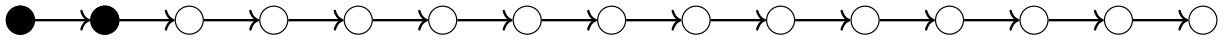

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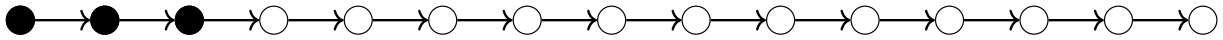

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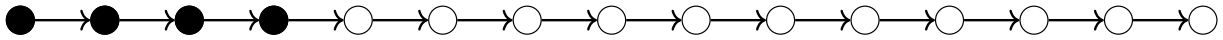

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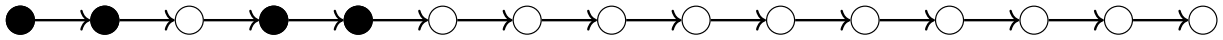

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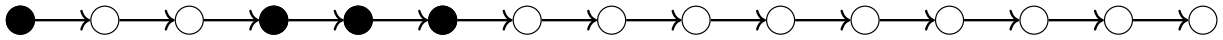

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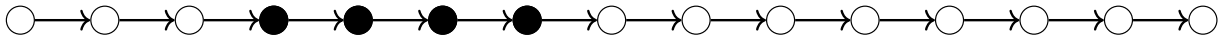
Parallel Reversible Pebbling [Our Work] 

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$\|$: parallel, $\leftrightarrows$: reversible

Our Result. For a line graph L_N with N nodes, we have $\mathsf{ST}^{\|,\leftrightarrows}(L_N) = \mathcal{O}\left(N^{1+\frac{2}{\sqrt{\log N}}}\right)$.

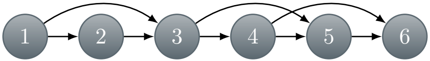
- We modified Li and Vitányi's (sequential) strategy [LV96]
- A similar (sequential) argument was implicitly assumed by Bennett [Ben89] but was not formalized as a reversible pebbling strategy

Reversible Pebbling Attack 2

Attack on Any (e, d)-Reducible DAGs

Definition. A DAG $G = (V, E)$ is *(e, d)-reducible* if there exists a *depth-reducing set* $S \subseteq V$ of size $|S| \leq e$ such that the longest path in $G - S$ has length $\leq d$.

Example. $(2, 2)$-reducible graph



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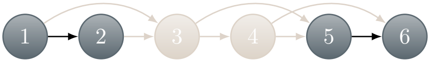

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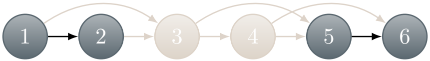

Our Result. If G is (e, d)-reducible, then $\mathsf{ST}^{\|,\Leftrightarrow}(G) = \mathcal{O}(Ne + Nd2^d)$.

- It becomes useful when $e \ll N$ and $d \ll \log N$ (which implies $\mathsf{ST}^{\|,\Leftrightarrow}(G) \ll \mathcal{O}(N^2)$)

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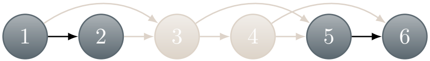

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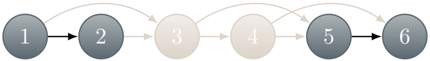

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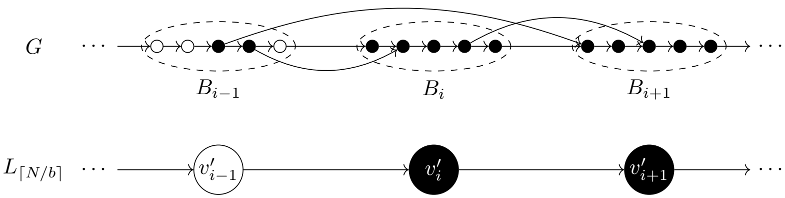
Reversible Pebbling Attack 3

Using an Induced Line Graph



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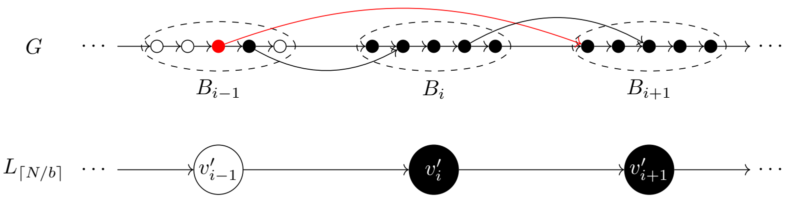

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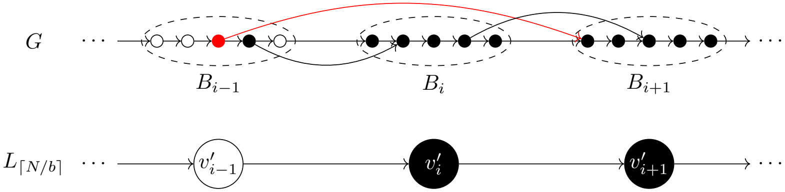

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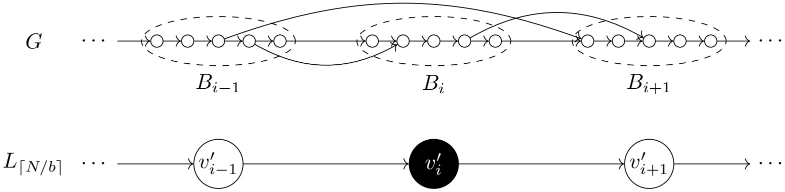

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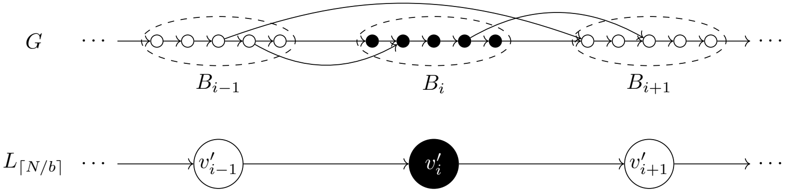

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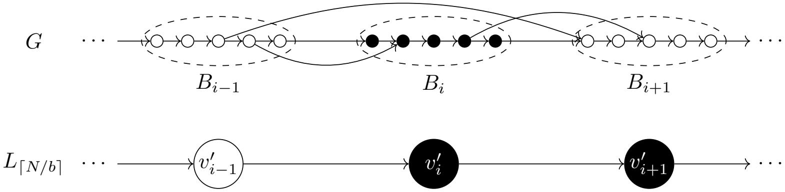

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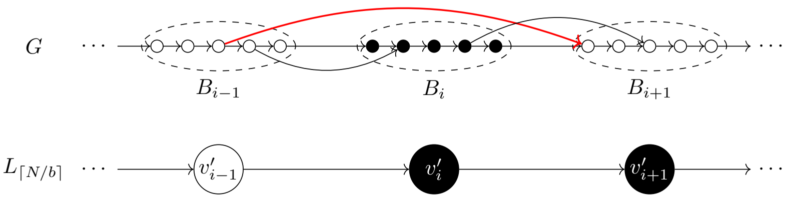

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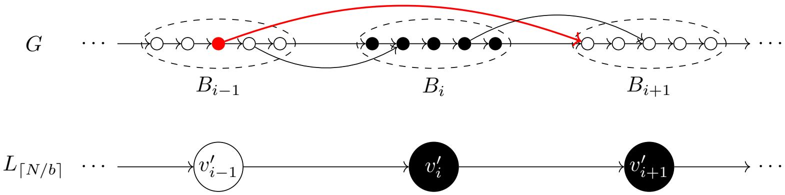

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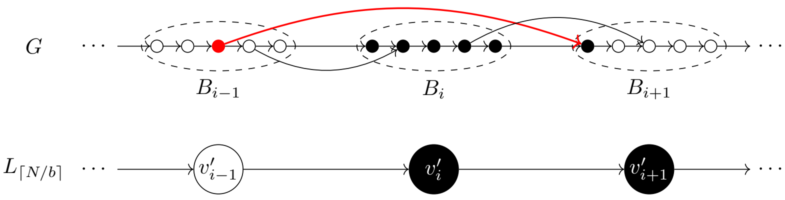

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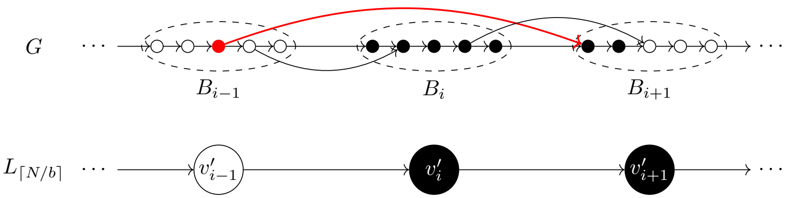

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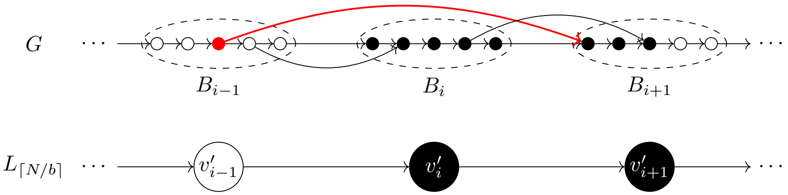

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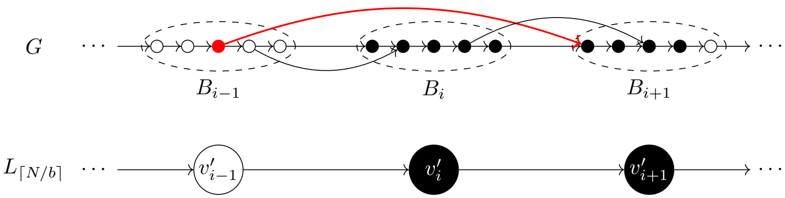

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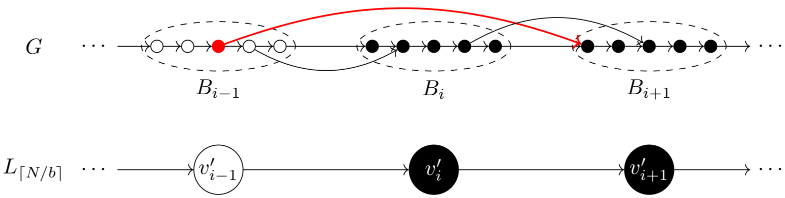

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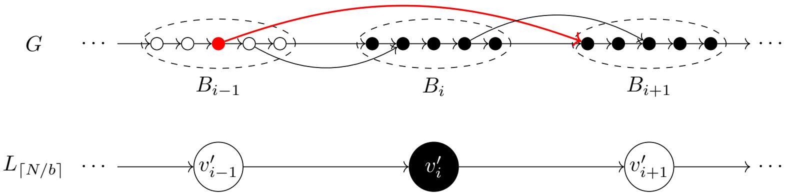

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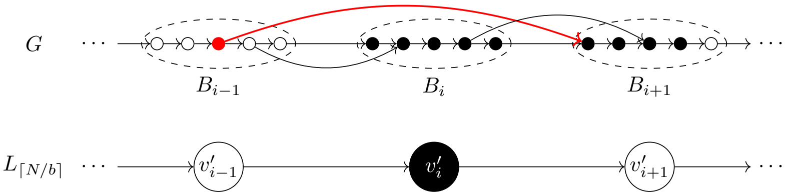

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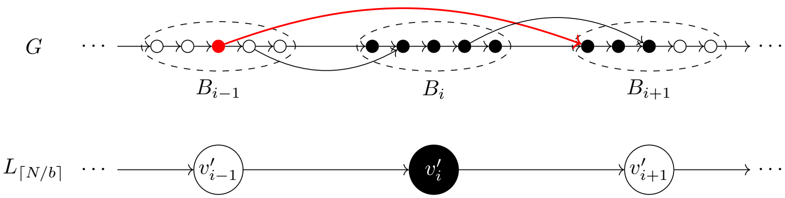

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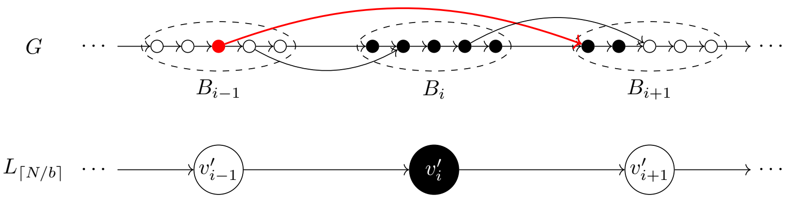

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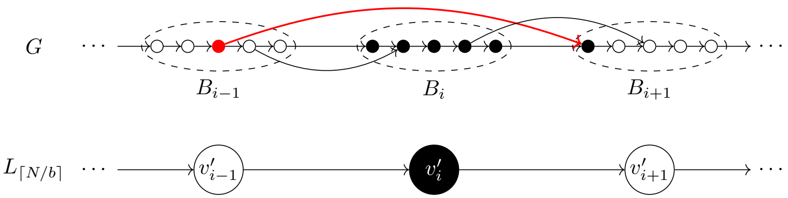

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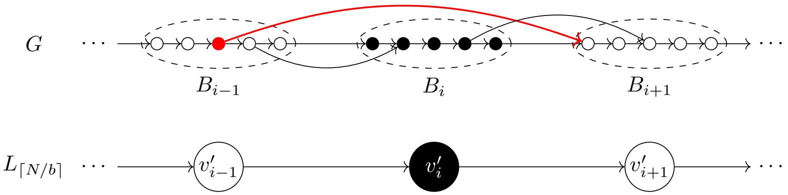

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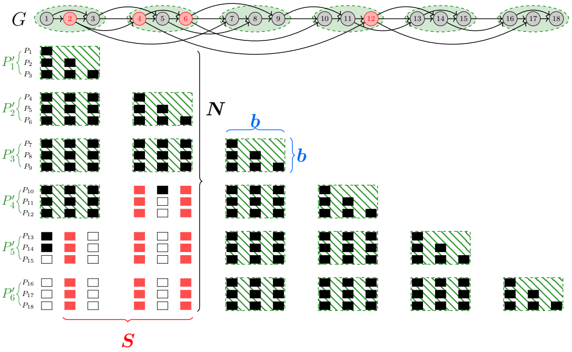

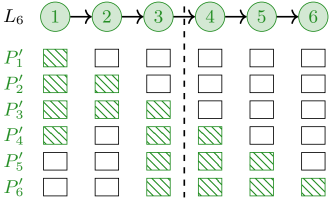

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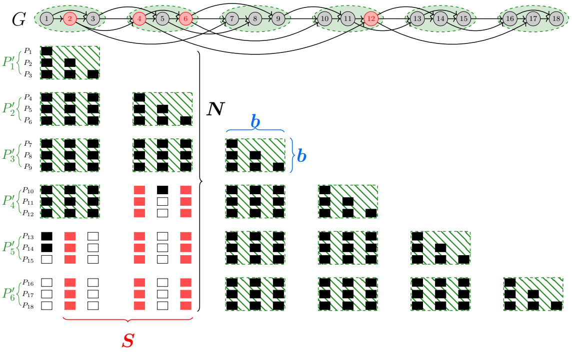

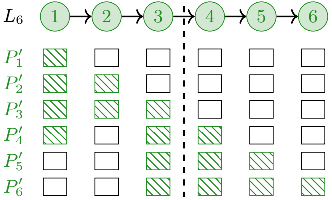

Our Result.

$\mathsf{ST}^{\|,\Leftrightarrow}(G) = \mathcal{O}\left(SN + b^2 \cdot \mathsf{ST}^{\|,\Leftrightarrow}(L_{\lceil N/b \rceil})\right)$, where S = (# skip nodes) and $b > 0$: block size

iMHF Example: DRSample

Attack Using an Induced Line Graph

- **DRSample** [ABH17]: a practical iMHF candidate with stronger *classical* memory-hardness
- For DRSample, we showed that (whp) the number of *skip nodes* is at most

$$(\text{\# skip nodes}) = \mathcal{O}\left(\frac{N \log\log N}{\log N}\right),$$

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- **Note.** DRSample admits a more efficient reversible pebbling attack than Argon2i-A/B

cf.) $\mathsf{ST}^{\parallel,\leftrightarrow}(\text{Argon2i-A}) = \mathcal{O}\left(\frac{N^2 \log\log N}{\sqrt{\log N}}\right)$ and $\mathsf{ST}^{\parallel,\leftrightarrow}(\text{Argon2i-B}) = \mathcal{O}\left(\frac{N^2}{\sqrt[3]{\log N}}\right)$

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Our Result: We extend the (non-reversible) Alwen and Blocki's attack [AB16]

Theorem. If G is (e, d)-reducible with N nodes with indegree δ, then

$$\mathsf{aST}^{\parallel,\leftrightarrow}(G) \leq \min_{g \geq d} \left\{ 2N \left(\frac{2Nd}{g} + e + (\delta + 1)g \right) + N + \frac{2Nd}{g} \right\}.$$

- **Corollary:** $\mathsf{aST}^{\parallel,\leftrightarrow}(\text{Argon2-A}) = \mathcal{O}(N^{1.75} \log N)$ and $\mathsf{aST}^{\parallel,\leftrightarrow}(\text{Argon2-B}) = \mathcal{O}(N^{1.8})$.

Conclusion

- We introduced the parallel reversible pebbling game, and
- We use this game to analyze the reversible space-time complexity of a line graph and data-independent Memory-Hard Functions such as Argon2i-A/B and DRSample
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  - Can we extend the recursive pebbling attack [ABP17] to the reversible setting?
- Is there a DAG with constant indegree having (parallel) reversible ST-cost $\Omega(N^2)$?
  - Candidate: DRS+BRG [BHK$^+$19], none of our attacks performed well against DRS+BRG
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  - We only showed that efficient reversible pebbling attacks yield efficient quantum pre-imag attacks, but not the reverse direction

References I

Joël Alwen and Jeremiah Blocki, *Efficiently computing data-independent memory-hard functions*, CRYPTO 2016, Part II (Matthew Robshaw and Jonathan Katz, eds.), LNCS, vol. 9815, Springer, Heidelberg, August 2016, pp. 241–271.

Joël Alwen, Jeremiah Blocki, and Ben Harsha, *Practical graphs for optimal side-channel resistant memory-hard functions*, ACM CCS 2017 (Bhavani M. Thuraisingham, David Evans, Tal Malkin, and Dongyan Xu, eds.), ACM Press, October / November 2017, pp. 1001–1017.

Joël Alwen, Jeremiah Blocki, and Krzysztof Pietrzak, *Depth-robust graphs and their cumulative memory complexity*, EUROCRYPT 2017, Part III (Jean-Sébastien Coron and Jesper Buus Nielsen, eds.), LNCS, vol. 10212, Springer, Heidelberg, April / May 2017, pp. 3–32.

Charles H. Bennett, Ethan Bernstein, Gilles Brassard, and Umesh V. Vazirani, *Strengths and weaknesses of quantum computing*, SIAM J. Comput. **26** (1997), no. 5, 1510–1523.

Charles H. Bennett, *Time/space trade-offs for reversible computation*, SIAM J. Comput. **18** (1989), no. 4, 766–776.

Jeremiah Blocki, Benjamin Harsha, Siteng Kang, Seunghoon Lee, Lu Xing, and Samson Zhou, *Data-independent memory hard functions: New attacks and stronger constructions*, CRYPTO 2019, Part II (Alexandra Boldyreva and Daniele Micciancio, eds.), LNCS, vol. 11693, Springer, Heidelberg, August 2019, pp. 573–607.

References II

Lov K. Grover, *A fast quantum mechanical algorithm for database search*, 28th ACM STOC, ACM Press, May 1996, pp. 212–219.

Burt Kaliski, *PKCS #5: Password-Based Cryptography Specification Version 2.0*, RFC 2898, RSA Laboratories, September 2000.

Richard Král'ovič, *Time and space complexity of reversible pebbling*, SOFSEM 2001: Theory and Practice of Informatics (Berlin, Heidelberg) (Leszek Pacholski and Peter Ružička, eds.), Springer Berlin Heidelberg, 2001, pp. 292–303.

Ming Li and Paul Vitányi, *Reversibility and adiabatic computation: Trading time and space for energy*, Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences **452** (1996), no. 1947, 769–789.

Giulia Meuli, Mathias Soeken, Martin Roetteler, Nikolaj Bjorner, and Giovanni De Micheli, *Reversible pebbling game for quantum memory management*, 2019 Design, Automation Test in Europe Conference Exhibition (DATE), 2019, pp. 288–291.

Niels Provos and David Mazières, *A future-adaptive password scheme*, Proceedings of the Annual Conference on USENIX Annual Technical Conference (USA), ATEC '99, USENIX Association, 1999, p. 32.